22. Konjugovano kompleksni broj

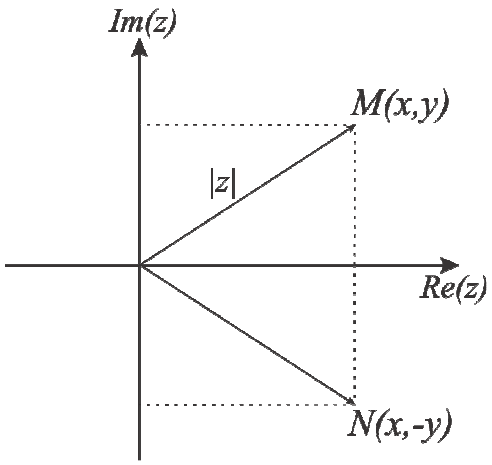
4° Za kompleksan broj $z = a + ib$ njemu konjugovano kompleksan broj je $\bar{z} = a - ib$.

Primer 1.

a) $z = 3 - i \Rightarrow \bar{z} = 3 + i$,

b) $z = 5 + 3i \Rightarrow \bar{z} = 5 - 3i$.

Geometrijski posmatrano, svakom kompleksnom broju $z = x + iy$ odgovara tačka $M(x, y)$ u pravouglom koordinatnom sistemu. Njemu konjugovan kompleksni broj ima koordinatu $N(x,-y)$ koja je simetrična tački M u odnosu na x-osu.



5° Modul kompleksnog broja $z = x + iy$ (intenzitet kompleksnog broja $z = x + iy$, udaljenost kompleksnog broja $z = x + iy$ od koordinatnog početka) je nenegativan broj $|z| = \sqrt{x^2 + y^2}$.

Primer 2. Ako je dat komplesan broj $z = 3 - 4i$ onda je njegov moduo

$$|z| = \sqrt{3^2 + (-4)^2} = \sqrt{25} = 5.$$

ZADATAK 1. Odredi module kompleksnih brojeva:

a) $z = 2 - i$,

b) $z = 2\sqrt{6} + 5i$,

c) $z = i$,

d) $z = 9 + 2i$,

e) $z = 3 + 2i\sqrt{2}$,

f) $z = \frac{15}{2} - \frac{35}{2} i$.

Rešenje:

a) $z = 2 - i \Rightarrow |z| = \sqrt{2^2 + (-1)^2} = \sqrt{5}$,

b) $z = 2\sqrt{6} + 5i \Rightarrow |z| = \sqrt{(2\sqrt{6})^2 + 5^2} = \sqrt{49} = 7$,

c) $z = i \Rightarrow |z| = \sqrt{0^2 + 1^2} = 1$,

d) $z = 9 + 2i \Rightarrow |z| = \sqrt{9^2 + 2^2} = \sqrt{85}$,

e) $z = 3 + 2i\sqrt{2} \Rightarrow |z| = \sqrt{3^2 + (2\sqrt{2})^2} = \sqrt{17}$,

f) $z = \frac{15}{2} - \frac{35}{2} i \Rightarrow |z| = \sqrt{\left(\frac{15}{2}\right)^2 + \left(-\frac{35}{2}\right)^2} = \sqrt{\frac{1450}{4}} = \frac{5}{2}\sqrt{58}$.

Za svaki kompleksan broj $z = a + ib$ važi:

a) $z + \bar{z} = 2\,\mathrm{Re}(z) = 2a$,

b) $z \cdot \bar{z} = |z|^2$,

c) $|z_1 \cdot z_2| = |z_1| \cdot |z_2|$,

d) $|z^2| = |z|^2$,

e) $\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$,

f) $\overline{z_1 \cdot z_2} = \bar{z}_1 \cdot \bar{z}_2$,

g) $|z| = |\bar{z}|$.

ZADATAK 2. Predstaviti sledeće brojeve u kompleksnoj ravni i odrediti im module:

a) $z = 2$,

b) $z = -1$,

c) $z = i$,

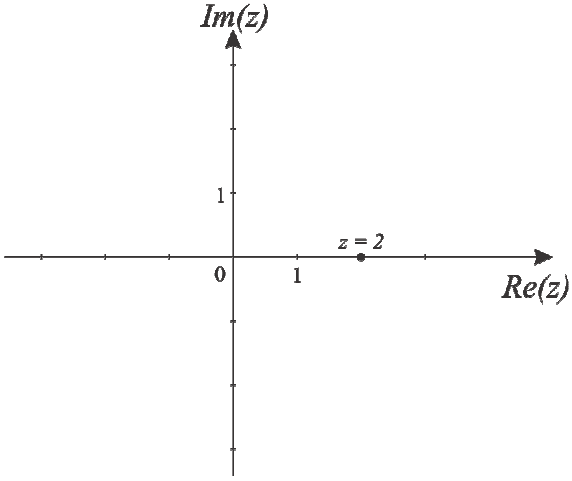
d) $z = -3i$,

e) $z = 1 + i$,

f) $z = 2 - 3i$,

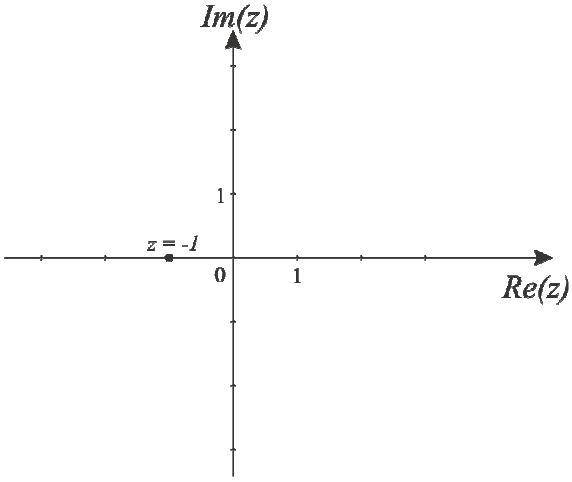
Rešenje:

a)



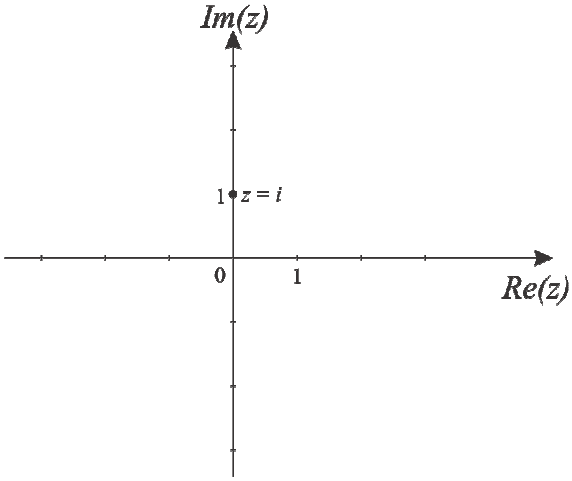
$$z = 2 \Rightarrow |z| = \sqrt{2^2 + 0^2} = 2,$$

b)



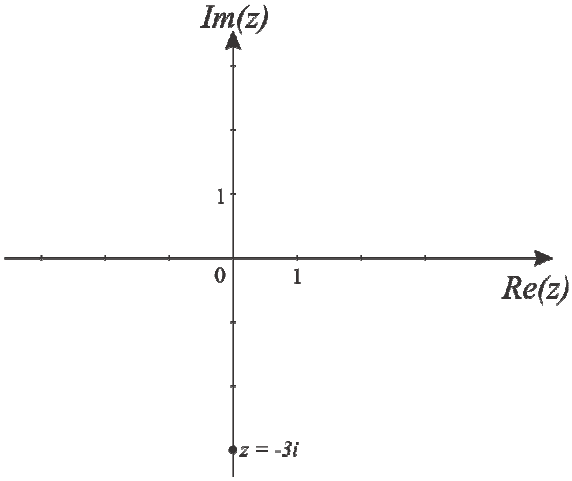
$$z = -1 \Rightarrow |z| = \sqrt{(-1)^2 + 0^2} = 1,$$

c)



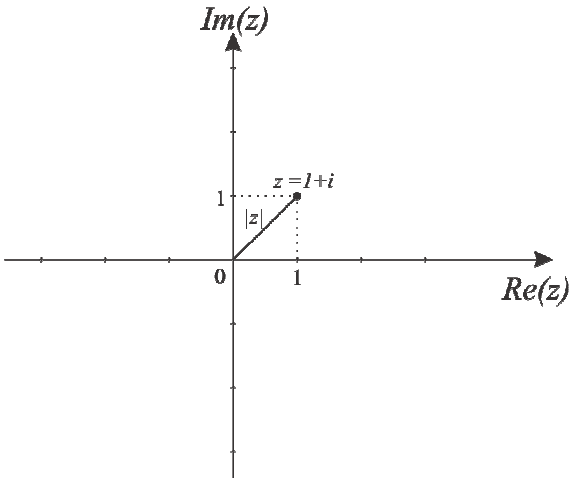
$$z = i \Rightarrow |z| = \sqrt{0^2 + 1^2} = 1,$$

d)



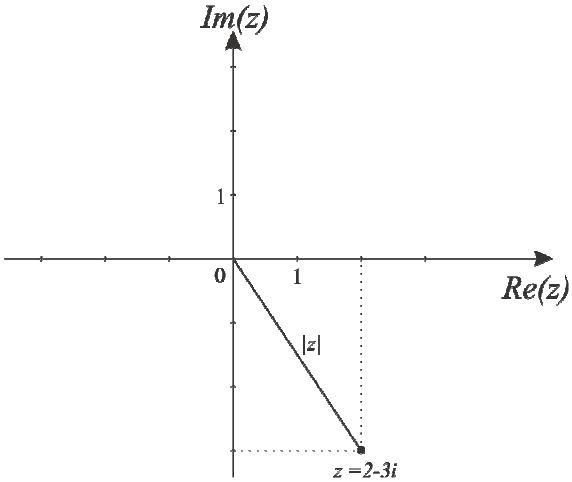
$$z = -3i \Rightarrow |z| = \sqrt{0^2 + (-3)^2} = 3,$$

e)



$$z = 1 + i \Rightarrow |z| = \sqrt{1^2 + 1^2} = \sqrt{2},$$

f)



$$z = 2 - 3i \Rightarrow |z| = \sqrt{2^2 + (-3)^2} = \sqrt{13}.$$

DOMAĆI ZADATAK: Vene T. Bogoslavov 2 – 384.