10. Proširivanje i skraćivanje korena

4^o Ako je $a \geq 0$ i $m, n \in N$ onda važi $\sqrt[n]{a} = \sqrt[m \cdot n]{a^m}$.

Primer 1.

a) $\sqrt[6]{5^3} = \sqrt[2 \cdot 3]{5^3} = \sqrt{5}$, ako ne piše koji je koren podrazumevana vrednost je 2;

b) $\sqrt[15]{7^5} = \sqrt[5 \cdot 3]{7^5} = \sqrt[3]{7}$;

c) $\sqrt[5]{7^{15}} = \sqrt[5]{7^{5 \cdot 3}} = 7^3$.

5^o Ako je $a \geq 0$ i $m, n \in N$ onda važi $\sqrt[m]{\sqrt[n]{a}} = \sqrt[m \cdot n]{a}$.

Primer 2.

a) $\sqrt{\sqrt[3]{25}} = \sqrt[2 \cdot 3]{25} \approx 1{,}71$;

b) $\sqrt[4]{\sqrt[6]{1000}} = \sqrt[2 \cdot 4]{10^3} = \sqrt[3 \cdot 8]{10^3} = \sqrt[8]{10}$;

6^o Ako su $a \geq 0$ i $b \geq 0$ i $n \in N$ onda važi $a\sqrt[n]{b} = \sqrt[n]{a^n b}$.

ZADATAK 1. Transformisati izraze (izvući činilac ispred korena u izrazima):

a) $\sqrt{50}$

b) $\sqrt{18}$

c) $\sqrt{75}$

d) $\sqrt{147}$

e) $\sqrt{125}$

Rešenje:

a) $\sqrt{50} = \sqrt{2 \cdot 25} = 5\sqrt{2}$;

b) $\sqrt{18} = \sqrt{2 \cdot 9} = 3\sqrt{2}$;

c) $\sqrt{75} = \sqrt{3 \cdot 25} = 5\sqrt{3}$;

d) $\sqrt{147} = \sqrt{3 \cdot 49} = 7\sqrt{3}$;

e) $\sqrt{125} = \sqrt{5 \cdot 25} = 5\sqrt{5}.$

ZADATAK 2. Uneti činilac pod koren u sledećim izrazima:

a) $3\sqrt{5}$

b) $\frac{1}{4}\sqrt{48}$

c) $-\frac{1}{7}\sqrt{245}$

d) $-0{,}1\sqrt{200}$

e) $5\sqrt{\frac{x}{5}}$

f) $\frac{1}{2}\sqrt{12a}$

Rešenje:

a) $3\sqrt{5} = \sqrt{3^2 \cdot 5} = \sqrt{45}$;

b) $\frac{1}{4}\sqrt{48} = \sqrt{\frac{1}{16} \cdot 48} = \sqrt{3}$;

c) $-\frac{1}{7}\sqrt{245} = -\sqrt{\frac{245}{49}} = -\sqrt{5}$;

d) $5\sqrt{\frac{x}{5}} = \sqrt{25 \cdot \frac{x}{5}} = \sqrt{5x}$;

e) $\frac{1}{2}\sqrt{12a} = \sqrt{\frac{12a}{4}} = \sqrt{3a}$.

ZADATAK 3. Transformisati izraze (izvući činilac ispred korena u izrazima):

a) $2x\sqrt{18a^5y^3}$

b) $a\sqrt{72a^3x^2y^3}$

c) $4ay^2\sqrt{2a^5x^2y^4}$

d) $\frac{2}{3x}\sqrt{\frac{27x^2}{8}}$

e) $\sqrt{\frac{9a^3}{4}+4a^3}$

Rešenje:

a) $2x\sqrt{18a^5y^3}=2x\sqrt{2\cdot 9\cdot a\cdot a^4\cdot y\cdot y^2}=2x\cdot 3\cdot a^2\cdot y\cdot\sqrt{2\cdot a\cdot y}=6a^2xy\sqrt{2ay}$;

b) $a\sqrt{72a^3x^2y^3}=a\sqrt{2\cdot 36\cdot a\cdot a^2\cdot x^2\cdot y\cdot y^2}=a\cdot 6\cdot a\cdot x\cdot y\cdot\sqrt{2\cdot a}=6a^2xy\sqrt{2ay}$;

c) $4ay^2\sqrt{2a^5x^2y^4}=4ay^2\sqrt{2\cdot a\cdot a^4\cdot x^2\cdot y^4}=4ay^2\cdot a^2\cdot x\cdot y^2\cdot\sqrt{2a}=4a^3xy^4\sqrt{2a}$;

d) $\frac{2}{3x}\sqrt{\frac{27x^2}{8}}=\frac{2}{3x}\sqrt{\frac{3\cdot 9\cdot x^2}{2\cdot 4}}=\frac{2}{3x}\cdot\frac{3x}{2}\sqrt{\frac{3}{2}}=\sqrt{\frac{3}{2}}$;

e) $\sqrt{\frac{9a^3}{4}+4a^3}=\sqrt{a^3\left(\frac{9}{4}+4\right)}=\sqrt{a\cdot a^2\cdot\frac{9+16}{4}}=a\sqrt{a\cdot\frac{25}{4}}=\frac{5a}{2}\sqrt{a}$;