13. Racionalisanje imenioca

U osnovnoj školi smo naučili racionalisaanje korena sa kvadratnim korenom, tj.

$$\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}.$$

Kod *n*-tog korena ideja je ista samo što moramo red stepena da dopunimo do reda korena. To znači da moramo obezbediti da nakon racionalizacije imamo $\sqrt[n]{a^n}$.

Primer 1. Racionalisati imenioc:

a) $\frac{1}{\sqrt[3]{2}} \cdot \frac{\sqrt[3]{2^2}}{\sqrt[3]{2^2}} = \frac{\sqrt[3]{4}}{\sqrt[3]{2} \cdot \sqrt[3]{2^2}} = \frac{\sqrt[3]{4}}{\sqrt[3]{2^3}} = \frac{\sqrt[3]{4}}{2}$;

b) $\frac{10}{\sqrt[4]{5}} \cdot \frac{\sqrt[4]{5^3}}{\sqrt[4]{5^3}} = \frac{10\sqrt[4]{5^3}}{\sqrt[4]{5} \cdot \sqrt[4]{5^3}} = \frac{10\sqrt[4]{5^3}}{\sqrt[4]{5^4}} = \frac{10\sqrt[4]{5^3}}{5} = 2\sqrt[4]{125}$;

c) $\frac{ab}{\sqrt[3]{a^2b}} \cdot \frac{\sqrt[3]{ab^2}}{\sqrt[3]{ab^2}} = \frac{ab\sqrt[3]{ab^2}}{\sqrt[3]{a^3b^3}} = \frac{ab\sqrt[3]{ab^2}}{ab} = \sqrt[3]{ab^2}$.

Ako u imeniocu imamo zbir ili razliku dva kvadratna korena, za postupak racionalizacije koristimo formulu razlike kvadrata:

$$A^2 - B^2 = (A-B)(A+B)$$

Primer 2. Racionalisati imenioc

a) $\frac{\sqrt{3}}{\sqrt{5}-\sqrt{2}}$

b) $\frac{\sqrt{a}}{\sqrt{a}+\sqrt{b}}$

Rešenje:

a) $\frac{\sqrt{3}}{\sqrt{5}-\sqrt{2}} = \frac{\sqrt{3}}{\sqrt{5}-\sqrt{2}} \cdot \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}} = \frac{\sqrt{3}(\sqrt{5}+\sqrt{2})}{(\sqrt{5})^2-(\sqrt{2})^2} = \frac{\sqrt{15}+\sqrt{6}}{5-2} = \frac{\sqrt{15}+\sqrt{6}}{3}$;

b) $\frac{\sqrt{a}}{\sqrt{a}+\sqrt{b}} = \frac{\sqrt{a}}{\sqrt{a}+\sqrt{b}} \cdot \frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \frac{\sqrt{a}(\sqrt{a}-\sqrt{b})}{(\sqrt{a})^2-(\sqrt{b})^2} = \frac{a-\sqrt{ab}}{a-b}, a \neq b.$

Ako u imaniocu imamo zbir ili razliku kubnih korena, koristimo formule za zbir ili razliku kubova:

$$A^3 - B^3 = (A - B)(A^2 + AB + B^2)$$
$$A^3 + B^3 = (A + B)(A^2 - AB + B^2)$$

Primer 3.

a) $\frac{1}{\sqrt[3]{3} + \sqrt[3]{2}}$

b) $\frac{5}{\sqrt[3]{5} - \sqrt[3]{4}}$

Rešenje:

a) $$\frac{1}{\sqrt[3]{3} + \sqrt[3]{2}} = \frac{1}{\sqrt[3]{3} + \sqrt[3]{2}} \cdot \frac{\sqrt[3]{3^2} - \sqrt[3]{3}\sqrt[3]{2} + \sqrt[3]{2^2}}{\sqrt[3]{3^2} - \sqrt[3]{3}\sqrt[3]{2} + \sqrt[3]{2^2}} = \frac{\sqrt[3]{9} - \sqrt[3]{3 \cdot 2} + \sqrt[3]{4}}{(\sqrt[3]{3})^3 + (\sqrt[3]{2})^3} = \frac{\sqrt[3]{9} - \sqrt[3]{6} + \sqrt[3]{4}}{5},$$

b) $$\frac{5}{\sqrt[3]{5} - \sqrt[3]{4}} = \frac{5}{\sqrt[3]{5} - \sqrt[3]{4}} \cdot \frac{\sqrt[3]{5^2} + \sqrt[3]{5} \cdot \sqrt[3]{4} + \sqrt[3]{4^2}}{\sqrt[3]{5^2} + \sqrt[3]{5} \cdot \sqrt[3]{4} + \sqrt[3]{4^2}} = \frac{5 \cdot (\sqrt[3]{25} + \sqrt[3]{5 \cdot 4} + \sqrt[3]{16})}{(\sqrt[3]{5})^3 - (\sqrt[3]{4})^3} =$$

$$= \frac{5 \cdot (\sqrt[3]{25} + \sqrt[3]{20} + \sqrt[3]{16})}{5 - 4} = 5 \cdot (\sqrt[3]{25} + \sqrt[3]{20} + \sqrt[3]{16}).$$

DOMAĆI ZADATAK: Racionalisati imenioce:

a) $\frac{14}{\sqrt{7}}$,

b) $\frac{2}{3\sqrt{5}}$,

c) $\frac{3}{\sqrt[3]{9}}$,

d) $\frac{xy}{\sqrt{x^2 y^3}}$,

e) $\frac{1}{\sqrt{7} - \sqrt{6}}$,

f) $\frac{3}{2+\sqrt{3}}$,

g) $\frac{1}{\sqrt[3]{3}+\sqrt[3]{2}}$,

h) $\frac{1}{\sqrt[3]{3}-\sqrt[3]{2}}$.